

**Dynamical Systems Theory and Lagrangian Data Assimilation in
Geophysical Fluid Dynamics
FY 2013 (October 1, 2012 – September 30, 2013)**

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LONG-TERM GOALS

To improve our understanding and predictive capability for three dimensional, time-dependent ocean circulation.

OBJECTIVES

Our main objective is to develop modeling approaches, theory and observational diagnostics that will allow a better understanding of the role of processes that are intrinsically three dimensional and time varying. We hope to use this knowledge to inform parameterizations and predictions, and also to guide observational strategies, including strategies for autonomous underwater sensors and vehicles.

APPROACH

Our approach centers on the Lagrangian view of ocean circulation; that is, the study of fluid motion through tracking of fluid trajectories. The methods of analysis are based in the mathematical field of dynamical systems.

WORK COMPLETED

We have developed a number of methods for identifying and computing internal boundaries and other coherent features that drift and evolve with ocean currents and that separate qualitatively different regions. These structures provide a template for understanding fluid pathways, mixing, and transport processes in 3D. These methods have been tested using a variety of models ranging from idealized to realistic. We have also developed diagnostics that indicate when two-dimensional analysis suffices. Another major thrust involves the development of methods for assimilating Lagrangian data into ocean models.

RESULTS

We note that our program was the subject of a 3-year review at ONR in May of this year. All presentations from that meeting can be downloaded from the our MURI program website:

<http://www.whoi.edu/ocean3dplus1/>

under MURI meetings.

New methods for computing barriers and other Lagrangian coherent structures (LCS) include the ergodicity defect (a measure of trajectory complexity), Koopman Operator techniques (which rely on properties measured along trajectories), techniques that identify regions of high hyperbolicity (corresponding to rapid fluid stretching and folding), and methods that calculate the rate of separation of fluid from its neighbors. Examples are depicted in Figures 1-5. All have been proved capable of identifying key Lagrangian structures in the flow field and the choice of a particular method will depend on the nature of the model or observation information available. We have also made progress in identifying the kind of barriers that can exist in fully 3D, time-dependent flows (Figures 5-7 and 9). Our search for barriers is being extended to more realistic 3D+1 simulations (various figures).

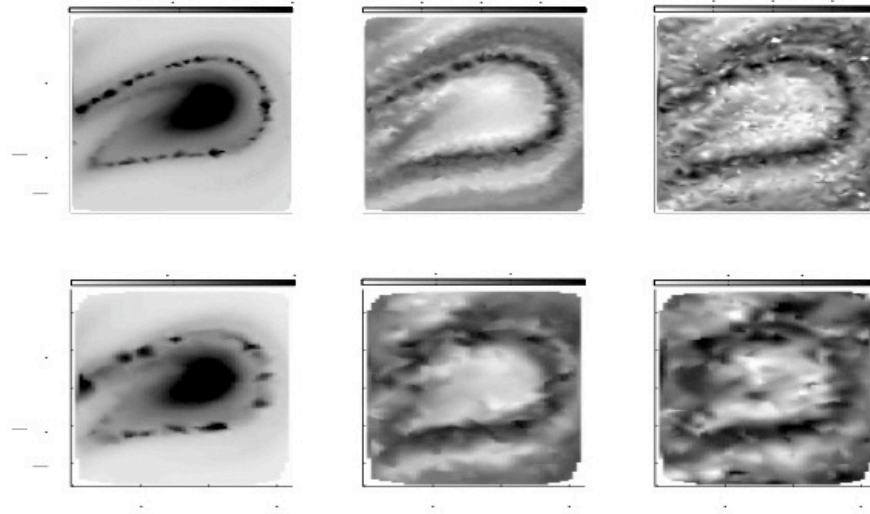


Figure 1. A method for calculation of LCS using measures of trajectory complexity: work by Rypina, Pratt, Scott, and Brown.

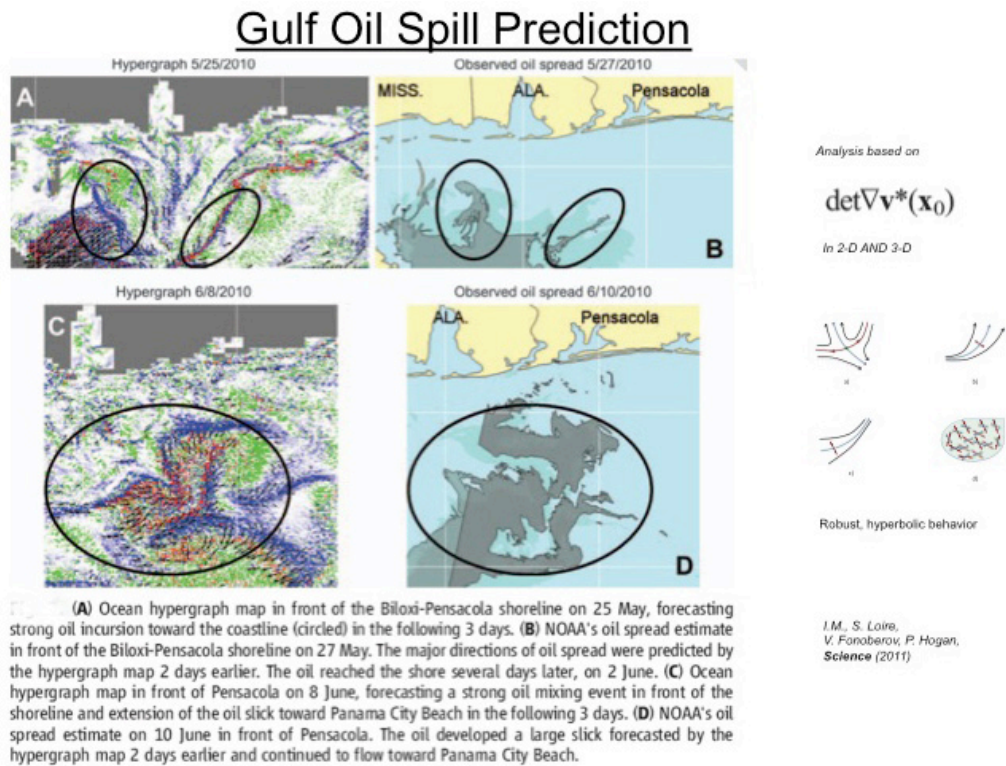


Figure 2. Mesohyperbolicity identifies areas of strong filamentation (Mezic).

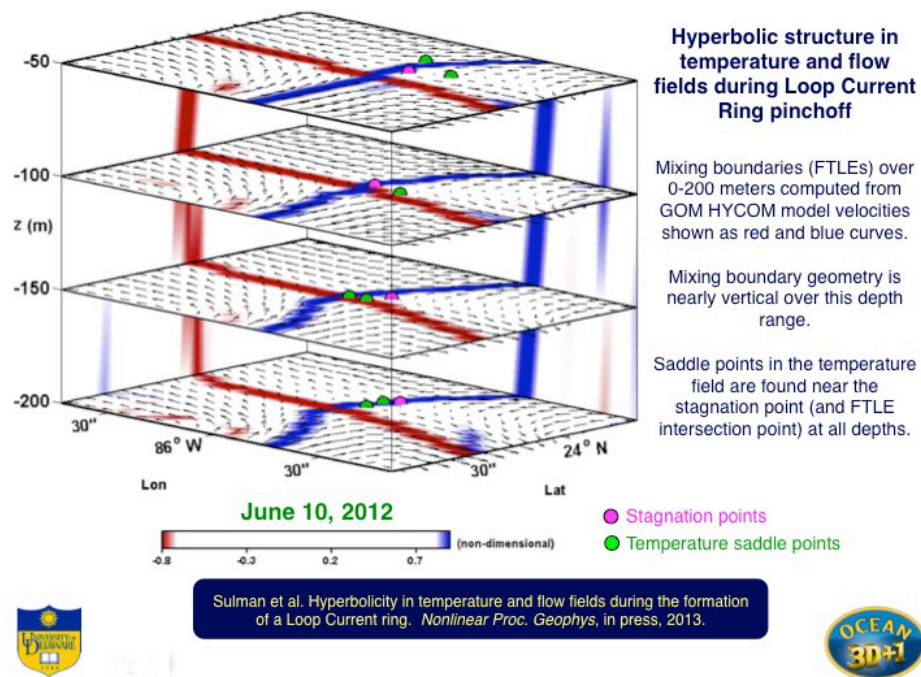


Figure 3. Use of mesohyperbolicity to identify barriers and strongly stirred regions.

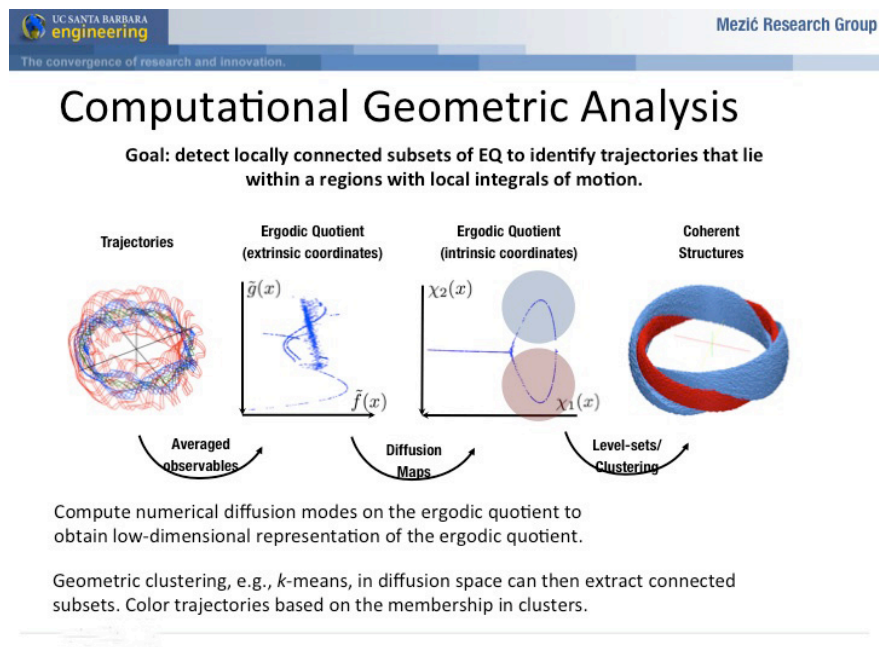


Figure 4. The Ergodic Quotient is based on Lagrangian averages of scalar quantities.

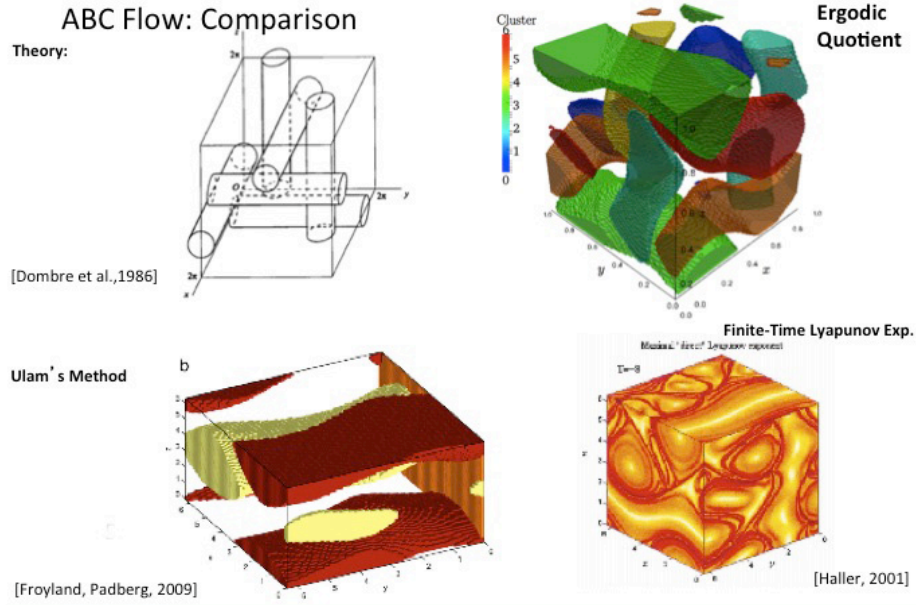


Figure 5. Comparison with known examples such as ABC flow is favorable.

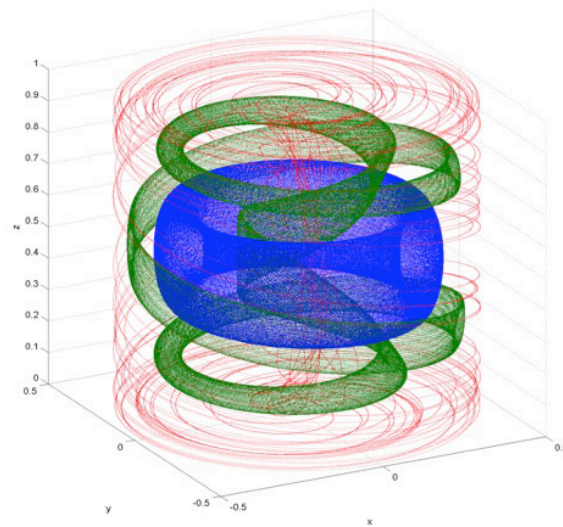


Figure 6. Idealized models of fully 3D eddy circulations tell us what Lagrangian barriers look like in 3D. In this case, the barriers consist of tori, including twisted hula hoops. Sandwiched between the surfaces are bands and islands of chaos. (From Pratt, Rypina, Ozgokmen, Wang, and Childs. JFM 2013.

Object resulting from double resonance

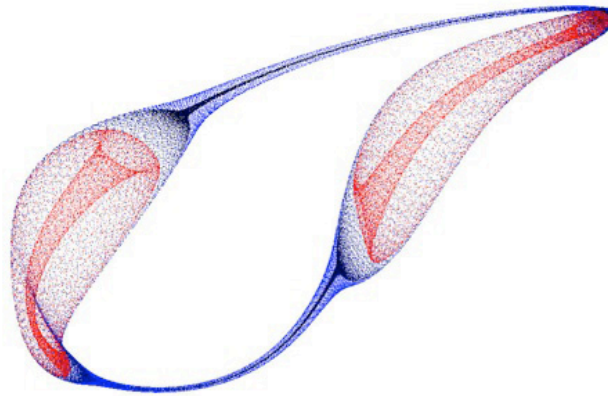


Figure 7: When the flow is fully 3D and time-dependent, the barriers become exotic. In this case the cocoon-like structure pulsates and rotates in the eddy.

Do the 3D FTLEs detect any coherent features in a highly turbulent situation?

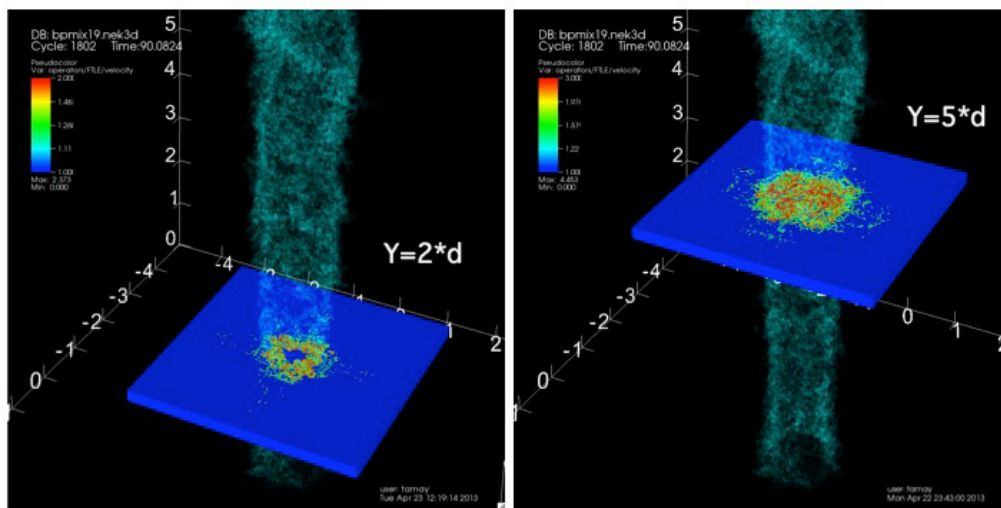


Figure 8: The methods used to compute LCS also provides information about changes in the stirring distribution in the cross section of a fully 3D turbulent rising oil plume. (Ozgokmen)

Our team is also exploring the application of LCS methodology in connection with other realistic models. This work emphasizes the role of the submesoscale (Figs. 10-14), which is commonly unresolved in ocean models.

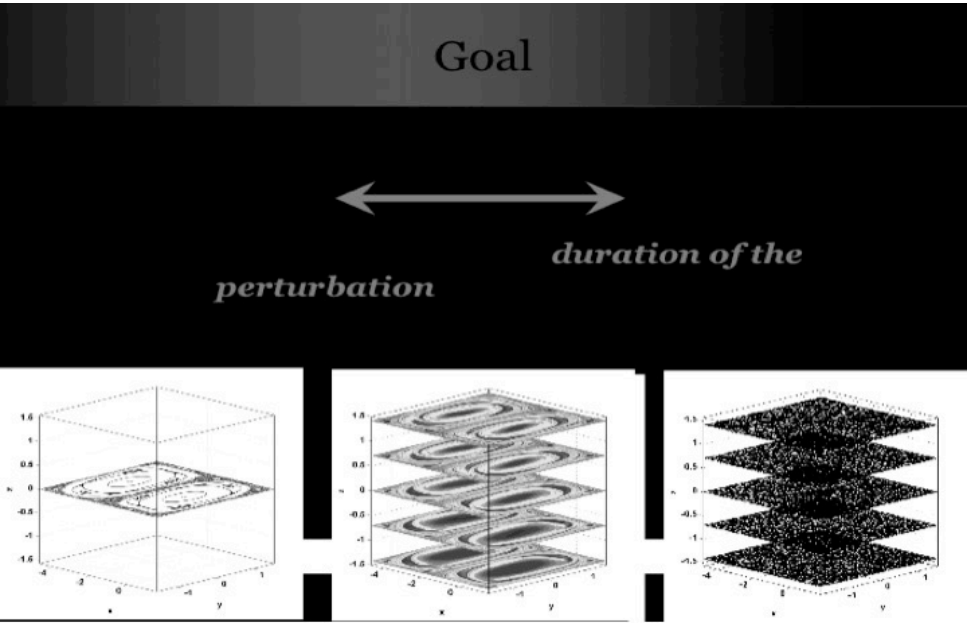
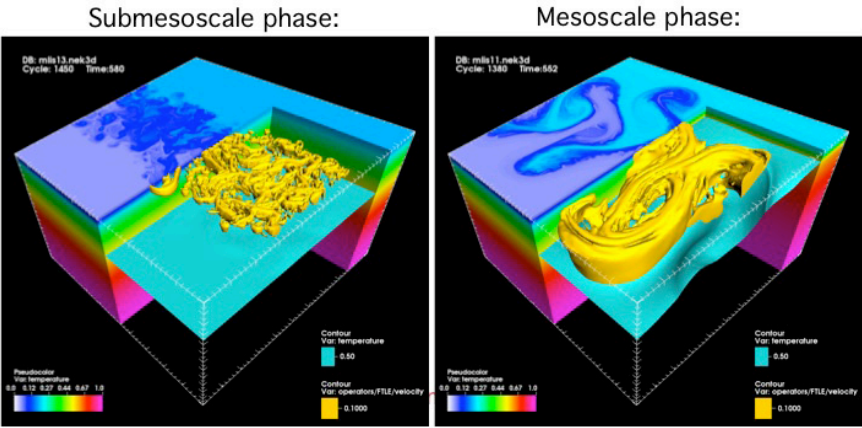


Figure 9. We can get a path of understanding into fully 3D, time dependent systems but looking a weakly 3D systems for long time intervals. (Llewellyn Smith)

3D FTLE:



Clearly different turbulent coherent structures:
shallow submesoscale eddies vs deep mesoscale features...

Figure 10. We are exploring the effects of submesoscale motions (left), which are unresolved in most models. (Ozgokmen)

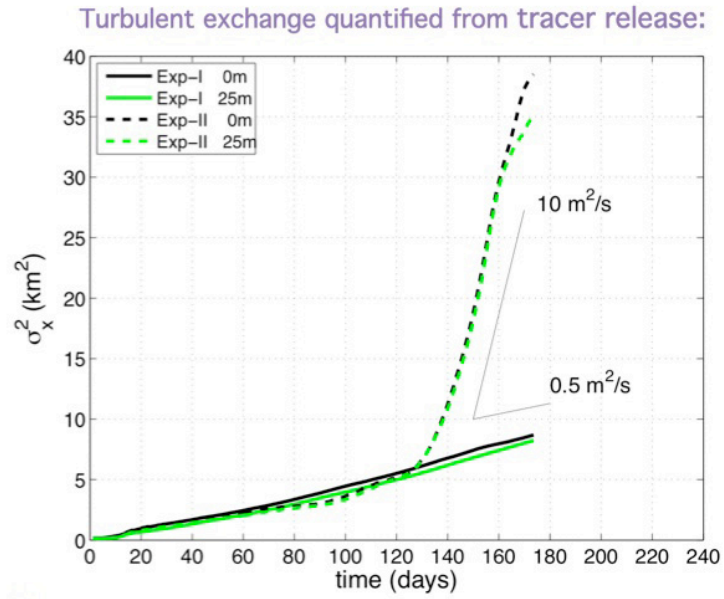


Figure 11. *An order of magnitude increase in frontal turbulent diffusivity after the deep instability kicks in; submesoscale overcome by mesoscale in this case.*

Cohabitation of Mesoscale and Submesoscale:

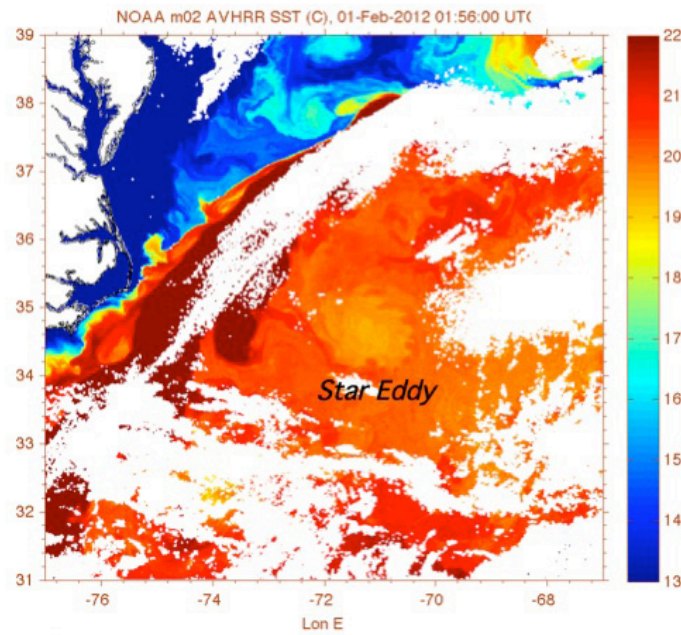


Figure 12. *The instabilities that cause star eddies reside in the submesoscale (Ozgokmen).*

3D FTLEs From LES of Oceanic Eddies:

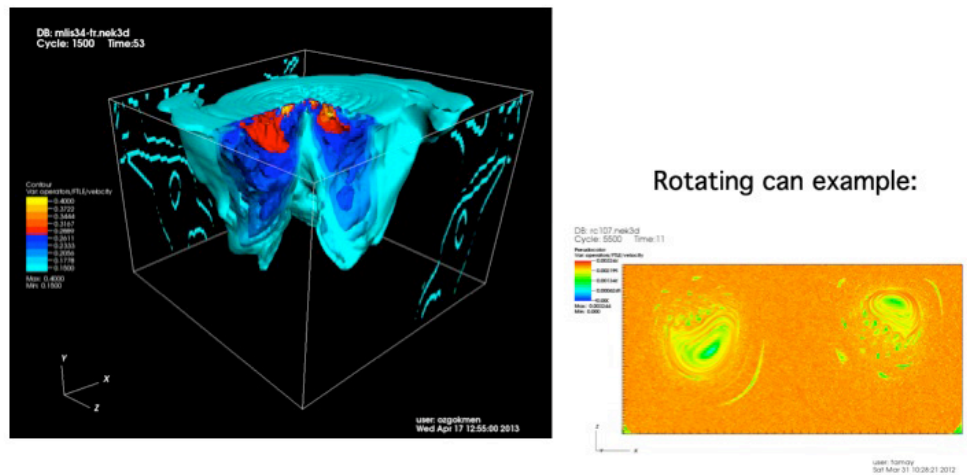
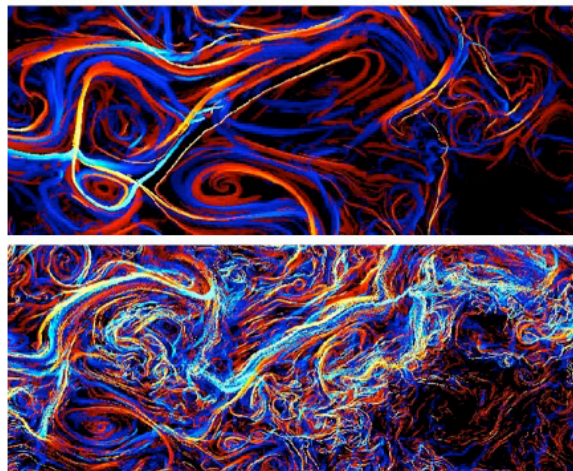


Figure 13. We are searching for barriers in realistic 3D, time-dependent eddies. Do we see generalizations of the tori found in idealized models? (Ozgokmen, Pratt, Rypina, and Wang.)

Which Scales Control Transport in the Ocean?

Poje et al. (2010)

Haza et al. (2012)



1/12 degree HYCOM

Image credit: Haza

1/48 degree HYCOM

- (I) Are the long-living, slow mesoscale features enough to compute transport?
- (II) Or, rapidly-evolving, smaller submesoscale transport barriers are needed?

Figure 14. Comparison between LCS are different model resolution. The lower panel resolves the submesoscale.

In the area of Lagrangian Data Assimilation (Fig. 15), we are developing methods that are more efficient and accurate, and that work in 3D. This work attempts to use information obtained from LCS analysis in order to determine where to best launch floats and drifters and to optimize navigation strategy for autonomous underwater vehicles Fig. 16.

Augmented system

Append equations for drifters (floats, gliders, AUVs)

Ide, Jones and Kuznetsov (2002)

$$\mathbf{x} = \begin{pmatrix} \mathbf{x} \\ \mathbf{x}_D \end{pmatrix} \quad \text{-- augmented state vector}$$

$$\frac{d\mathbf{x}_F^f}{dt} = M_F(\mathbf{x}_F^f, t) \quad \text{-- flow equations}$$

$$\frac{d\mathbf{x}_D^f}{dt} = M_D(\mathbf{x}_D^f, \mathbf{x}_F^f, t) \quad \text{-- advection equation}$$

Apply filtering to augmented system:

1. Ensemble Kalman Filter
2. MCMC (Particle Filter)

Figure 15. *The general ideal behind Lagrangian data assimilation work of Jones, Spiller and colleagues.*

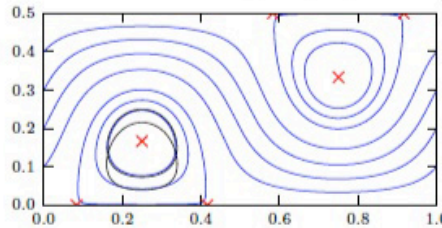
Instrument control to optimize information

$$\frac{\partial v}{\partial t} = 0$$

$$\frac{dz}{dt} = v(z) + f(z)$$

Add *simple* control:

$$f(z) = (\varsigma, 0)$$



$$\begin{aligned} y_k^1 &= \mathbf{x}(t_k) + \eta_k, & \eta_k &\sim \mathcal{N}(0, \sigma^2 I), & k &= 1, \dots, K/2 & \text{no control} \\ y_k^2 &= \mathbf{x}(t_k) + \eta_k, & \eta_k &\sim \mathcal{N}(0, \sigma^2 I), & k &= K/2, \dots, K & \text{control} \end{aligned}$$

Want: $P(v | y^1, y^2)$

Figure 16. *Is it possible to optimize navigation strategies for AUV's in order to collect the best data for LDA? (Jones, Spiller and colleagues.)*

IMPACT/APPLICATIONS

The Lagrangian view offers insights into transport and mixing processes that are different from those gained in the Eulerian frame. These insights naturally lead to better physics-based parameterizations, such as turbulent eddy coefficients. (One of the criticisms of Eulerian-based parameterizations is that they depend only on the local properties of the flow and do not consider patches of turbulence that are advected into the local.) Lagrangian methods can also identify time-dependent barriers that may be important in predicting regions that pollutants, including oil and radioactive material, have difficulty crossing.

The Lagrangian approach to data assimilation will, in principle, allow information that has been previously discarded to be used in prediction and state estimation. In particular, previous assimilation schemes that use sequences velocity measurements from drifting sensors do not assimilate the information that each measurement comes from the same trajectory.

TRANSITIONS

The tools being developed will aid in the prediction of catastrophic release of oil, radioactive material or floating debris into the ocean.

RELATED PROJECTS

A British Petroleum-funded project on the Gulf of Mexico oil spill. Prof. Tamay Ozgokmen is involved along with several other PIs.

PUBLICATIONS

- Amey, C., B. Sundaram, and A. C. Poje. Mixing in mixed phase spaces. To be published in *Physics of Fluids*, May 2013.
- Apte, A., and C.K.R.T. Jones, 2013. The impact of nonlinearity on Lagrangian data assimilation. *Nonlinear Processes in Geophysics*, 20, 329-341.
- Apte, A., E. Spiller, and C.K.R.T. Jones. Assimilating en-route Lagrangian observations. *Tellus A* [accepted, refereed].
- Balasuriya, S., G. Froyland, and N. Santitissadeekorn, 2014. Absolute flux optimising curves of flows on a surface. *Journal of Mathematical Analysis and Applications*, **409**, 119-139.
- Budišić, Marko, and Igor Mezić, 2012. Geometry of the ergodic quotient reveals coherent structures in flows. *Physica D*, **241**(15), 1255-1269, 10.1016/j.physd.2012.04.006.
- Chabreyrie, R., and S. G. Llewellyn Smith. Short- and Long-Time Structures in a Three-Dimensional Time Dependent Flow. Submitted to *Physics of Fluids*.
- Cox, G. Large-time uniqueness in a data assimilation problem for Burgers' equation, 2012. *Inverse Problems*. <http://arxiv.org/abs/1207.4782v1>.
- Cox, G. Uniqueness and non-uniqueness of minimizers in variational data assimilation. Submitted to *Physica D*.
- Cox, G. The L^2 essential spectrum of the 2D Euler operator. Submitted to *Journal of Mathematical Fluid Mechanics*.
- Froyland, G., C. Horenkamp, V. Rossi, N. Santitissadeekorn, and A. Sen Gupta, 2012. Three-dimensional characterization and tracking of an Agulhas Ring. *Ocean Modelling*, **52–53**, pg. 69–75.
- Griffa, A., A. C. Haza, T. M. Özgökmen, A. Molcard, V. Taillandier, K. Schroeder, Y. Chang, and P.M. Poulain, 2013. Investigating transport pathways in the ocean. *Deep Sea Research II*, **85**, 81-95.
- Haza, A. C., T. M. Özgökmen, A. Griffa, Z. D. Garraffo, and L. Piterbarg, 2012. Parameterization of particle transport at submesoscales in the Gulf Stream region using Lagrangian subgridscale models. *Ocean Modelling*, **42**, 31-49.
- Huntley, H. S., B. L. Lipphardt, Jr., and A. D. Kirwan, Jr., 2011. Surface Drift Predictions of the *Deepwater Horizon* Spill: The Lagrangian Perspective. *Geophysical Monograph 195: Monitoring and Modeling the Deepwater Oil Spill: A Record-Breaking Enterprise*, pg. 179-195.
- Loire, S., P. Kauffmann, I. Mezić and C. D. Meinhart, 2012. A theoretical and experimental study of AC electrothermal flows. *J. Phys. D: Appl. Phys.*, **45**, 185301.
- Mensa, J., A. Griffa, Z. Garraffo, T.M. Özgökmen and M. Veneziani: Seasonality of the submesoscale dynamics in the Gulf Stream region. *Ocean Dynamics*, submitted: March 2013.

- Özgökmen, T.M., A.C. Poje, P.F. Fischer, H. Childs, H. Krishnan, C. Garth, A. Haza, and E. Ryan, 2012. On multi-scale dispersion under the influence of surface mixed layer instabilities and deep flows. *Ocean Modelling*, **56**, 16-30 DOI: 10.1016/j.ocemod.2012.07.004.
- Özgökmen, T. M., A. C. Poje, P. F. Fischer, and A. C. Haza. Large eddy simulations of mixed layer instabilities and sampling strategies, 2011. *Ocean Modelling*, **39**(3-4), 311-331, DOI: 10.1016/j.ocemod.2011.05.006.
- Piterbarg, L.I., V. Taillandier, and A. Griffa. Investigating frontal variability from repeated glider transects in the Ligurian Current (North West Mediterranean Sea). *J. Mar. Sys.*, submitted (April 2, 2013).
- Pratt, L. J., I. Rypina, T. M. Özgökmen, and Y. Bebieva. Chaotic advection in a rotating-cylinder flow. Submitted to *J. Fluid Mech.*
- Pratt, L., I. Rypina, T.M. Özgökmen, H. Childs, and Y. Bebieva. Chaotic advection in a steady, three-dimensional, Ekman-driven eddy. *Journal of Fluid Mechanics*, in revision, March 2013.
- Rypina, I., S. E. Scott, L. J. Pratt, and M. G. Brown, 2011. Investigating the connection between complexity of isolated trajectories and Lagrangian coherent structures. *Nonlin. Processes Geophys.*, **18**, 977-987.
- Rypina, I.I, L. Pratt, and I. Mezic. Resonance phenomena in 3D time-dependent volume-preserving flows with symmetries. In preparation to *Nonlin. Processes Geophys.*, Special Issue on "Nonlinear Processes in Oceanic and Atmospheric Flows".
- Santitissadeekorn, N., E.T. Spiller, C.K.R.T. Jones, R. Rutarindwa, L. Liu, and K. Ide. Observing System Simulation Experiments of Cross-Layer Lagrangian Data Assimilation. Submitted to *Dynamics of Atmospheres and Oceans*, January, 2013.
- Schroeder, K., J. Chiggiato, A. C. Haza, A. Griffa, T. M. Özgökmen, P. Zanasca, A. Molcard, M. Borghini, P.M. Poulain, R. Gerin, Z. Zambianchi, P. Falco, and C. Trees, 2012. Targeted Lagrangian sampling of submesoscale dispersion at a coastal frontal zone. *Geophys. Res. Lett.*, **39**, L11608, doi.10.1029/2012GL051879.
- Sulman, M. H. M., H. S. Huntley, B. L. Lipphardt, Jr., and A. D. Kirwan, Jr, 2012. Out of Flatland: Three-Dimensional Aspects of Lagrangian Transport in Geophysical Fluids. *Geophysical Monograph 200: Lagrangian Modeling of the Atmosphere*, p. 77-84.
- Sulman, M. H. M., H. S. Huntley, B. L. Lipphardt Jr., and A. D. Kirwan Jr., 2013. Leaving Flatland: Diagnostics for Lagrangian coherent structures in three-dimensional flows. *Physica D*, **258**: 77-92.
- Sulman, M. H. M., H. S. Huntley, B. L. Lipphardt Jr., G. Jacobs, P. Hogan, and A. D. Kirwan Jr., 2013. Hyperbolicity in temperature and flow fields during the formation of a Loop Current ring. *Nonlinear Proc. Geophys.* [in press]
- Valentine, D. A., I. Mezić, Senka Maćešić, Nelida Črnjarić-Žić, Stefan Ivić, P. J. Hogan, V. A. doi: 10.1073/pnas.1108820109 water hydrocarbon irruption. *Proceedings of the National Academy of Sciences*, doi: 10.1073/pnas.1108820109.